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INVESTIGATION OF HEAT TRANSFER IN A GRADIENT FLOW REGION FOR PLANE TURBULENT JET IMPINGING ON A PLATE SITUATED NORMAL TO THE FLOW

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ABSTRACT. Experimental determination of the boundary conditions required for the solution system of equations of motion, discontinuity equation, and energy equation describing heat transfer in the gradient flow region arising when a plane isothermal turbulent jet impinges on a plate situated normal to the flow. The boundary conditions are obtained in the form of a universal relation between the velocity at the outer boundary of the boundary layer, the coordinate along the plate, and the spacing between the plate and the nozzle. Formulas for calculating the heat transfer coefficient in a laminar boundary layer are derived. A substantial discrepancy is found to exist between the experimental and theoretical results. An analysis of the changes in the experimental heat transfer coefficient near the spreading line as a function of the spacing between the nozzle and the plate, and of the influence of this spacing on the degree of turbulence indicates that the relation between this coefficient and the degree of turbulence may be considered to be linear in the first approximation. This result is used as a basis for deriving formulas for the heat transfer coefficient in the gradient flow region under consideration.

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When a flat jet impinges on a plate situated normal to the jet flow, /631*
and in the case of an axially symmetric jet [1], three regions of flow can be distinguished:

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*Numbers in the margin indicate pagination in the original foreign text.

1. A region of gradient flow where the pressure falls off along the plate from a maximum value at the spreading line to a value near atmospheric pressure at a certain distance x from the line. The velocity at the outer boundary of the boundary layer, U, increases from 0 at the spreading line to a maximum value, U_* at a distance x_* from it. The y-axis lies in the plane of symmetry of the jet normal to the plate, the x axis along the plate, normal to the spreading line.

Within this region beyond the limits of the boundary layer, the flow is considered potential and is described by the equation

$$U\frac{\partial U}{\partial x}=-\frac{1}{\rho}\frac{\partial p}{\partial x}. \tag{1}$$

2. A region of transition flow, where the velocity U remains practically constant.

3. A region of principal flow, where the velocity U begins to decrease as a result of the deceleration of the jet along the wall, and the pressure remains practically constant.

In the present paper, an investigation is made of only the gradient flow region. From the practical point of view this region is of the greatest importance. At the same time, the physical processes occurring here are the most complicated, and their analysis is very laborious.

Considering that the Reynolds numbers for the region in question are small, we assume that the boundary layer here is laminar, especially since $dP/dx < 0$.

Under these conditions, the system of differential equations of motion, continuity, energy, and the boundary conditions which describe the process of heat transfer can be written as

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+\nu\frac{\partial^2 u}{\partial y^2},\qquad (2)$$ /632

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0,\qquad (3)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=a\frac{\partial^2 T}{\partial y^2}.\qquad (4)$$

The boundary conditions are:

at $y=0$ $u=v=0$, $\overline{T}=0$;

at $y=\delta$ $u=U$, at $y=\delta_t$ $\overline{T}=1$.

The velocity at the outer boundary of the boundary layer can be represented by the first two terms of a series, which is in satisfactory agreement with the actual conditions (Figure 1).

$$U=\beta_1 x+\beta_2 x^3+\dots,\qquad (5)$$

where β_1, β_2 are constants dependent on $\bar{h}$. The equation of continuity is satisfied for

$$u=\frac{\partial\Psi}{\partial y},\quad v=-\frac{\partial\Psi}{\partial x}.\qquad (6)$$

Let us introduce a dimensionless coordinate $\eta=y\sqrt{\frac{\beta_1}{\nu}}$ and write the stream function Ψ as follows:

$$\Psi=\sqrt{\frac{\nu}{\beta_1}}\left[\beta_1 x f_1(\eta)+4\beta_3 x^3 f_3(\eta)+\dots\right],\qquad (7)$$

where $f_i(\eta)$ are functions of the dimensionless coordinate η.

On the basis of (6) and (7) we shall find the distribution of the velocities u and v in terms of the width of the boundary

$$u=\beta_1 f_1'(\eta)\,x+4\beta_3 f_3'(\eta)\,x^3+\dots,\qquad (8)$$

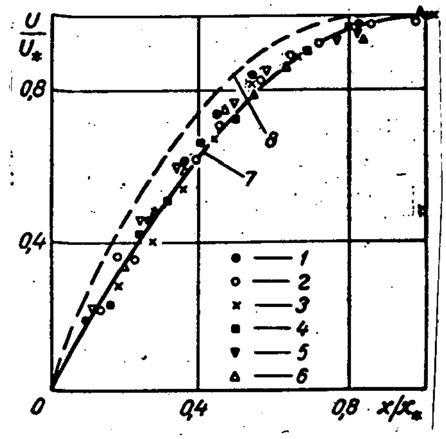

Figure 1. Velocity change at the outer border of the boundary layer: b_0 = 31 mm; u_0 = 15.5 m/sec; 1 - $\bar{h}$ = 1.67; 2 - 4; b_0 = 22 mm; u_0 = 17 m/sec; 3 - $\bar{h}$ = 6.25; 4 - 9.1; b_0 = 11 mm; u_0 = 20.8 m/sec; 5 - $\bar{h}$ = 17.8; 6 - 22.3; 7 - by Formula (15); 8 - based on the data of [6].

$$v = -\sqrt{\frac{\nu}{\beta_1}}\,[\beta_1 f_1(\eta) + 12\beta_3 f_3(\eta)\,x^2 + \ldots], \tag{9}$$

where the prime signifies differentiation with respect to η. The distribution of the dimensionless temperature across the boundary layer can be represented by [4] /633

$$\bar{T}(x, y) = \frac{T - T_w}{T_\infty - T_w} = F_0(\eta) + \frac{\beta_3}{\beta_1} x^2 F_2(\eta) + \ldots, \tag{10}$$

where $F_0(\eta)$, $F_2(\eta)$ are functions of the dimensionless coordinate η and Pr.

Substituting (8) and (9) in (2), and (10) in (4), after some algebra, we obtain the following system of differential equations [4, 7]:

$$\begin{aligned}
&f_1'^2 - f_1 f_1'' = 1 + f_1''', \\
&4f_1' f_3' - 3f_1'' f_3 - f_1 f_3'' = 1 + f_3''', \\
&\ldots\ldots\ldots\ldots\ldots\ldots \\
&\frac{1}{\Pr} F_0'' + f_1 F_0' = 0, \\
&\frac{1}{\Pr} F_2'' + f_1 F_2' - 2f_1' F_2 = -12 f_3 F_0', \\
&\ldots\ldots\ldots\ldots\ldots\ldots
\end{aligned} \tag{11}$$

with the boundary conditions of the form

from $\eta \to 0$ $f_1 = f_1' = f_3 = f_3' = 0, \quad F_0 = F_2 = 0, \quad F_4 = 0, \quad f_5 = f_5' = 0;$

from $\eta \to \infty$ $f_1' = 1, \quad f_3' = \frac{1}{4}, \quad F_0 = 1, \quad F_2 = 0, \quad F_4 = 0, \quad f_5' = \frac{1}{6}.$

The system of Equations (11) ith the above boundary conditions has been solved using numerical methods [2, 4].

The values of the unknown quantities f_i, f'_i, f''_i, f'''_i are given in [3], and F'_i in [4]. The heat flux to the wall can be found using Fourier's and Newton's law. Equating the absolute values of the right-hand sides, we obtain

$$\lambda\left(\frac{\partial T}{\partial y}\right)_{y=0}=\alpha(T_\infty-T_w), \qquad (12)$$

whence in view of (10) we find(1)

$$\alpha=\lambda\sqrt{\frac{\beta_1}{\nu}}\left[F'_0(0)+\frac{\beta_3}{\beta_1}F'_2(0)x^2+\ldots\right]. \qquad (13)$$

If the Prandtl number is Pr = 0.7, the values of the functions will be $F'_0(0)$ = 0.4959, $F'_2(0)$ = 0.4476 [4]. Taking these values into account, we shall represent (13) in the following form:

$$\mathrm{Nu}_x=b_0\sqrt{\frac{\beta_1}{\nu}}\left(0{,}496+0{,}448x^2\frac{\beta_3}{\beta_1}+\ldots\right). \qquad (14)$$

Formula (14) can be used in theoretical calculations assuming we know the values of the constants β_1 and β_3. These values have been determined experimentally. The experiments were done using nozzles with a slit 150 mm long and 5, 11, 20, 31 mm wide. A polished plate was 200 x 700 mm in size. The openings used to determine the static pressure were 0.3 mm in diameter, and the distance between them was 5 mm. The total pressure in the boundary /634
layer was measured using a pressure-measuring tube built especially for this purpose. The experiments were done with the stream velocities at the nozzle slit ranging from 5 to 25 m/sec.

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(1) Retaining the first two terms of the series.

The experimental results can be represented in the form of the following universal relationship (Figure 1):

$$\frac{U}{U_*} = 1{,}6\frac{x}{x_*} - 0{,}6\left(\frac{x}{x_*}\right)^3. \tag{15}$$

The quantities U_* and x_* depend on the distance between the nozzle slit and the plate. They can be determined from the experimental results by means of the following formulas:

for $1 \leq \bar{h} \leq 6.5$

$$\frac{U_*}{U_0} = \frac{1}{\bar{h}^{0,1}}; \quad \bar{x}_* = \frac{x_*}{b_0} = 1{,}7\bar{h}^{0,1}, \tag{16}$$

for $\bar{h} \geq 6.5$

$$\frac{U_*}{U_0} = \frac{2{,}3}{\bar{h}^{0,5}}; \quad \bar{x}_* = \frac{x_*}{b_0} = 0{,}58\bar{h}^{0,7}. \tag{17}$$

Comparing (5) and (14), we find that within the experimental range investigated

$$\beta_1 = 1{,}6\frac{U_*}{x_*}, \quad \beta_3 = 0{,}6\frac{U_*}{x_*^3}. \tag{18}$$

The theoretical relationships for the local values of the heat transfer coefficients α in the gradient flow region can be found by substituting (18) in (13), and taking (16) and (17) into account, i.e.,

for $1 \leq \bar{h} \leq 6.5$

$$\mathrm{Nu}_x = 0{,}48\,\mathrm{Re}_0^{0,5}\left(1 - 0{,}116\frac{\bar{x}^2}{\bar{h}^{0,2}}\right)\frac{1}{\bar{h}^{0,1}}, \tag{19}$$

for $\bar{h} \geq 6.5$

$$\mathrm{Nu}_x = 1{,}25\,\mathrm{Re}_0^{0,5}\left(1 - 1{,}05\frac{\bar{x}^2}{\bar{h}^{1,4}}\right)\frac{1}{\bar{h}^{0,6}}. \tag{20}$$

At the spreading line for $\bar{x} = 0$, Equations (19) and (20) will become:

for $1 \leq \bar{h} \leq 6.5$

$$Nu_0 = 0{,}48\, Re_0^{0,5} \bar{h}^{-0,1}, \tag{21}$$

for $\bar{h} \geq 6.5$

$$Nu_0 = 1{,}25\, Re_0^{0,5} \bar{h}^{-0,6}. \tag{22}$$

Comparison of the values of the heat transfer coefficient α, as calculated using Equations (21) and (22), with the values obtained directly from experiment [6] shows that

1. For $\bar{h} < 3$ one observes a good agreement between these values. The validity of the comparison is confirmed by a good agreement between the two relationships $\bar{U} = f(\bar{x})$ (Figure 1). The first was obtained experimentally in the present paper, and the second was obtained theoretically from Equation (1) on the basis of data on the distribution of pressure with $\bar{x}$, obtained from the papers by Gardon and Akfirat [5]. /635

We note that in the region $\bar{h} < 14$, there is a certain scatter among the experimental values, depending on the slit width b_0. The slit width b_0 affects the initial degree of turbulence of a jet, ε_0. Smaller values of α correspond to a lower degree of initial turbulence of the jet.

In order to make a comparison with an identical initial degree of turbulence, the theoretical values of α (21) were compared with the smallest values obtained experimentally.

2. For $\bar{h} \geq 3$ the experimental values of α become greater than the theoretical values. As $\bar{h}$ increases, the discrepancy between the experimental and theoretical values of the heat transfer coefficient becomes greater and attains a maximum at $\bar{h} \approx 14$.

3. With a further increase in $\bar{h} > 14$, the discrepancy remains the same.

The experimental values of the heat transfer coefficient in the neighborhood of the spreading line, α_0' for $\bar{h} > 14$ turn out to be approximately two times greater than the theoretical values of α_0. In order to understand this large discrepancy, let us consider the plots shown in Figure 2,a and b.

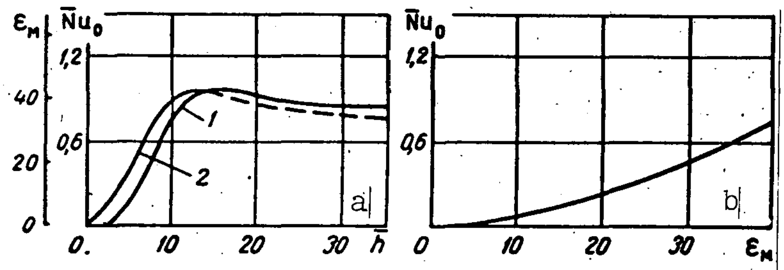

Figure 2. Effect of turbulence on heat transfer intensification at the spreading line:

a - 1 - Nu_0 versus $\bar{h}$; 2 - ε_M, % versus $\bar{h}$ [6];

b - Nu_0 versus ε_M.

Figure 2,a contains two plots. The first (1) is a plot of $\overline{Nu}_0 = f(\bar{h})$ at the spreading line, referred to the theoretical value α_0, (20) - (21) versus the distance of the plate from the nozzle. The second (2) is a plot of $\varepsilon_M = f(\bar{h})$ [5] of the local axial degree of turbulence of a jet versus the distance from the nozzle, where the dashed portion of Curve 2 was plotted on the basis of the fact that, for $\bar{h} > 14$, the authors found that ε_M remains approximately constant. These two plots were obtained for the same nozzle with $Re_0 = 5500$. The value of Nu_0 was found from

$$\overline{Nu}_0 = \frac{Nu_0'}{Nu_0} - 1, \tag{23}$$

The good correlation between Curves 1 and 2 in Figure 2,a permits us to draw the following conclusions:

1. The increase in the heat transfer coefficient α_0' at the plate with the distance from the nozzle is related to the degree of turbulence, ε_M, of the incoming jet.

2. To a first approximation, this relationship can be assumed to be linear.

Figure 2,b is a plot of $\overline{Nu}_0 = f(\varepsilon_M)$ which was obtained by replotting the curves shown in Figure 2,a.

On the basis of the relationships plotted in Figure 2,a and b, and assuming that the correction of the formulas for the effect of the degree of turbulence in the neighborhood of the spreading line is valid throughout the entire region of gradient flow, we obtain corrected formulas in the form: /636

for $1 \leq \bar{h} \leq 6.5$

$$Nu_x = 0{,}48\frac{Re_0^{0,5}}{\bar{h}^{0,1}}\left(1-0{,}116\frac{\bar{x}^2}{\bar{h}^{0,2}}\right)(1+0{,}015\varepsilon_M), \tag{24}$$

for $6.5 \leq \bar{h} \leq 12$

$$Nu_x = 1{,}25\frac{Re_0^{0,5}}{\bar{h}^{0,6}}\left(1-1{,}05\frac{\bar{x}^2}{\bar{h}^{1,4}}\right)(1+0{,}019\varepsilon_M), \tag{25}$$

for $\bar{h} > 12$

$$Nu_x = 1{,}25\frac{Re_0^{0,5}}{\bar{h}^{0,6}}\left(1-1{,}05\frac{\bar{x}^2}{\bar{h}^{1,4}}\right)(1+0{,}025\varepsilon_M). \tag{26}$$

The results based on Formulas (24) - (26) are plotted in Figure 3,a,b as solid curves; the experimental results — as dashed curves. Comparing the theoretical values of the heat transfer coefficient with the experimental ones, we can draw the following conclusion. Regardless of the fact that Formulas (24) - (26) were corrected for a single value of the Reynolds number $Re_0 = 5500$, the theoretical values are in satisfactory agreement with the experimental

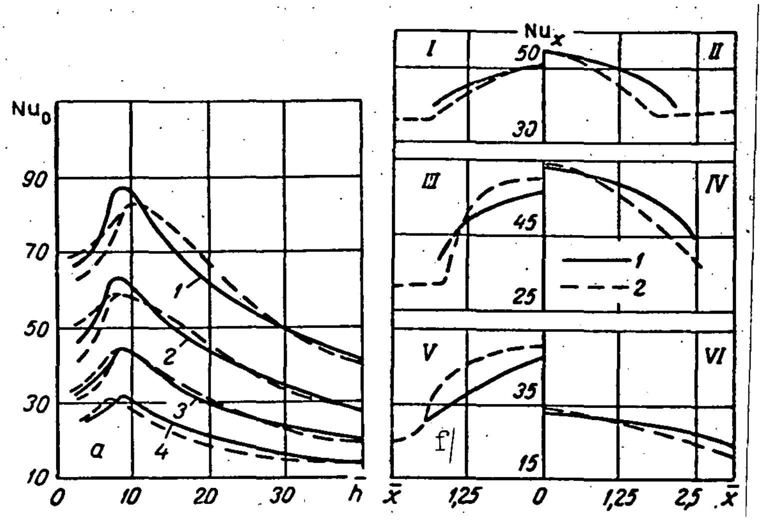

Figure 3. Heat transfer coefficients in gradient flow region:

a - Nu_0 versus $\bar{h}$ (1 - 4 (solid curves) based on Formulas (24) - (26); 1 - Re_0 = 22,000; 2 - 11,000; 3 - 5500; 4 - 2750); dashed curves are based on the experimental data of [5]; b - Nu versus $\bar{x}$ (Re_0 = 11,000; I - $\bar{h}$ = 2; II - 5; III - 6; IV - 8; V - 16; VI - 32; 1 - based on formulas (24) - (26); 2 - based on [5].

ones (discrepancy does not exceed 15%) within a wider range of Reynolds number (up to Re_0 = 22,000) and for nozzles with the initial degree of turbulence up to ε_0 = 7%.

Notation

b_o, width of nozzle slit; h, distance from nozzle cut to plate; $\bar{h}$, the same in dimensionless form; x, current abscissa; $\bar{x}$, dimensionless abscissa; $\bar{x}_*$, dimensionless abscissa when the velocity at outer border of wall boundary layer gets its maximum value; U, velocity at outer border of boundary layer; U_*, maximum velocity at outer border; $\bar{U}_*$, the same in dimensionless form; T_w, temperature of plate surface; T_∞, temperature of incoming flow; $\bar{T}$, dimensionless temperature; p, static pressure at given section of boundary layer; δ, thickness of dynamic boundary layer; δ_T, thickness of thermal boundary layer; α, heat transfer coefficient; Re_0, Reynolds number referred to parameters at nozzle cut; Nu, Nusselt number; ε_0, degree of initial turbulence; ε_M, degree of turbulence at jet axis referred to axial velocity.

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